Multi-model and multi-scale optimization strategies

Application to sonic boom reduction


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RÉSUMÉ. L’optimisation de la forme d’un avion supersonique nécessite un modèle composite comportant une composante 3D haute fidélité en mécanique des fluides et un modèle simplifié de propagation du bang. La prise en compte de cette complexité est étudiée dans le cadre d’une boucle d’optimisation, avec des adjoints discrets exacts de l’écoulement 3D et un système de déformation de maillage. L’introduction d’une méthode d’adaptation de maillage est aussi considérée.

Abstract. The shape optimization of a supersonic aircraft need a composite model combining a 3D CFD high-fidelity model and a simplified boom propagation model. The management of this complexity is studied in an optimization loop, with exact discrete adjoints of 3D flow and mesh deformation system. The introduction of a mesh adaptation algorithm is also considered.

MOTS-CLÉS : Optimisation, forme optimale, paramétrisation, différentiation automatique, adaptation de maillage anisotrope, bang sonique

KEYWORDS: Optimization, shape design, parameterization, automatic differentiation, anisotropic mesh adaptation, sonic boom.

1. Introduction

When passing to real life applications, optimal design algorithms have their complexity increased from many standpoints. We consider in this paper the treatment of multiple scales. By multiple scales, we mean that the modeled physics is not easily put in a unique PDE model, due to the very large and very small scales involved in the problem. A classical approach in that case consists in using specialized models for the different range of scales and considering the system as a multi-model one. Then the Multi-disciplinary Design Optimization (MDO) methods can be applied. These methods are able to couple different physics and also different level of fidelity for the same physics, see for example (Sobieszczanski-Sobieski et al., 1997) and (Alexandrov, 1997).

In our particular application, the sonic boom reduction, we address a part of the multi-scale issue by applying a mesh adaptation loop. The three main ingredients of mesh adaptation are:

(i) the choice of a criterion governing the local repartition of nodes and the alignments of elements,
(ii) the model of mesh,
(iii) the way the mesh generator is coupled with the rest of the algorithm.

Concerning (i), the user has to known what is the priority between working with accurate flow field evaluation or only with an accurate final answer, the optimal control approximation. Accuracy of the whole flow can be set in general and is solved by local error estimators, as interpolation estimators, see (Alauzet et al., 2007) and (Zhu et al., 1997). When focusing on functional value accuracy, goal-oriented a posteriori estimators are then considered, see (Giles et al., 2001) and (Venditti et al., 2002). Functional error can be also minimized as in (Koobus et al., 2007). The issue of accuracy in optimal control itself is addressed for example in (Polak, 1997) and (Becker et al., 2001).

The model of mesh, (ii), states for any kind of mesh modification algorithm, going from basic scheme such as mesh enrichment with patterns to global mesh modifications or generation with advanced meshing techniques. In our case, the volume mesh is adapted by local mesh modifications of the previous mesh using mesh operations such as vertex insertion, edge and face swap, collapse and node displacement. The vertex insertion procedure uses an anisotropic generalization of the Delaunay kernel. Coupling mesh adaptation with optimization algorithm, (iii), is not a trivial issue. Accuracy of adaptation should not be deteriorated by the optimization and robustness of optimization should not be lost by mesh updates. Our proposed approach is inspired by an analogy with the fixed point mesh adaptation method for unsteady flows proposed in (Alauzet et al., 2007).

An important feature of the study is the option of a gradient-based minimization. It is motivated by the necessity of considering a large number of design variables. The main cons of gradient-based minimization involve the risk of finding a local mini-
imum of poor quality, but more importantly, gradient-based methods need a differentiable functional. This requirement may interfere with the treatment of multiple physic scales. Two other important ingredients may have to be installed in the optimization loop. CAD shape parameterization belong to the functional to minimize, together with the process generating a new mesh for a new shape. Both have to be handled in the gradient evaluation.

Let us go a little further with the physics. Our application, sonic boom reduction, needs to handle the difficult problem of modeling sonic boom propagation. The system of Euler equations is recognized as a satisfying model for the whole physical process. But when passing to discrete model for numerical simulations, we have to discretize on a unique mesh small and large scales. Their ratio may be enormous. Indeed, shock width is in micron and the computational domain should extend from the aircraft to the ground, 15 kilometers below. Simpler models than Euler equations may be chosen for the whole physical process but they may loose some accuracy in the sonic boom prediction. An alternative is to use the Euler 3D model only in the vicinity of the aircraft. Then, a propagation model is utilized from Euler outputs and solves important features of pressure perturbation propagation down to the ground. It remains to determine the place of this extra model into the optimization process. The propagation could be directly included into the optimization loop as in (Farhat et al., 2002). Or, the propagation model could be treated separately by specifying the matching between the two models in the objective functional, see (Alonso et al., 2002). Regarding this study, the last option has been considered here.

In this paper, we first introduce the problem under study in terms of the basic high-fidelity model. Secondly, we present the two extensions of the basic model, i.e., the propagation model and mesh adaptive model. Thirdly, we describe a numerical optimization platform and show which kind of results can be obtained in combination with the propagation model. Then, preliminary experiments on an optimization platform coupled with mesh adaptation are described.

2. Mathematical design problem

2.1. Continuous standpoint

The flow around a supersonic aircraft is modeled by the Euler equations. Assuming that the gas is perfect, non viscous and that there is no thermal diffusion, the Euler equations for mass, momentum and energy conservation read:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{U}) &= 0, \\
\frac{\partial (\rho \vec{U})}{\partial t} + \nabla \cdot (\rho \vec{U} \otimes \vec{U}) + \nabla p &= 0, \\
\frac{\partial \rho E}{\partial t} + \nabla \cdot ((\rho E + p) \vec{U}) &= 0,
\end{align*}
\]
where $\rho$ denotes the density, $\vec{U}$ the velocity vector, $E = e + \frac{\|\vec{U}\|^2}{2}$ the total energy and $p = (\gamma - 1)\rho e$ the pressure with $\gamma = 1.4$ the ratio of specific heats and $e$ the internal energy. These equations are symbolically rewritten:

$$\frac{\partial W}{\partial t} + \nabla \cdot F(W) = 0,$$

where $W = (\rho, \rho u, \rho v, \rho w, \rho E)$ is the conservative variables vector and the vector $F$ represents the convective flux. In fact we are interested only in the steady solution of this system. It has to be evaluated from the region around the aircraft (near field) to the ground level (far field), so that the computational domain boundary involves the two components, viz. the aircraft wall and the ground. Slip conditions are assumed for both components. The pressure perturbation created by the supersonic aircraft flight extends down to the ground and is the source of uncomfortable noise for populations, i.e., the sonic boom. During this propagation to ground, the perturbation signal transforms, see Figure 1, left.

The optimization problem under study in this paper is to find an aircraft shape that would reduce the impact of this noise. This can be expressed as the research of a minimum of an objective functional measuring the deviation between the ground pressure signature and a target one. We define it in short as:

$$j = \int (p(W) - p_{\text{target}})^2 ds$$

where the integral is taken on ground surface and the functional $j$ depends on the aircraft shape through flow state $W$.

### 2.2. Numerical issues

Predicting the Euler flow down to the ground needs the resolution by the 3D mesh of wide range of scale sizes. Sharp shocks have to be solved with small numerical width, not necessary as small as micron, but yet of the order of centimeter, and this in a domain of tens of kilometers. As a consequence, three-dimensional Euler modeling of the pressure perturbation is not possible with standard meshes and today’s computers.

In the literature, authors starting with an accurate model -as Euler equations- restrict the 3D Euler computation to a near field sub-region of the domain, usually measured with the ratio of the diameter $R$ of the near field domain to the aircraft chord-length $L$. Then, propagation is handled with a simplified model. The accuracy of a composite model combining 3D Euler at near field and a propagation model depends on:

- the accuracy of the 3D Euler computation,
- the propagation model quality,
- and the adequacy between the matching location and simplification assumptions used in the propagation model.
In this paper, the 3D Euler equations are solved with a Finite Volume technique on unstructured tetrahedral meshes. The considered standard propagation model is the one proposed by Thomas (Thomas, 1972). Typically, matching distances to couple both models are set between $R/L = 0.5$ and $R/L = 5$. In order to address the problem of space-scale stiffness and to have a suitable adequacy between both models, we propose to apply an anisotropic mesh adaptation. Indeed, anisotropic mesh adaptation allows the solution to be propagated accurately in the near field domain. The \textit{a priori} handicap to handle is that the functional to minimize in the discrete optimization problem is changed as soon as mesh is changed.

Lastly, using the mesh adaptation does not dispense to use also Euler and propagation together, and an important question arises:

\textit{How to handle the composite model Euler/Propagation inside the optimization loop?}

To address this point, two main strategies can be considered:

– the propagation model is put into the state equation of the Optimal Control problem, see for example (Farhat \textit{et al.}, 2002). The difficulty may come from a lack of differentiability of the propagation model.

– the propagation model is used for building a mean field target pressure, see for example (Alonso \textit{et al.}, 2002). Then we need, for searching a particular ground signature, to solve an inverse problem providing the mean field target pressure. A difficulty occurs from the lack of invertibility of the propagation model. Nonetheless, we still consider this option.

These two extensions, propagation and mesh adaptation models, from the basic numerical model are now examined in more details.

3. Propagation model

3.1. Atmospheric pressure wave propagation

The propagation code uses a ray tracing algorithm based upon the waveform parameter method developed in (Thomas, 1972). It allows us to propagate near field perturbations to the ground in order to get the sonic boom signature of the aircraft. In this approach, the pressure wave is characterized by three parameters (see Figure 1):

– $m_i$ the slope of pressure waveform segment $i$

– $\Delta p_i$ the pressure rise across shock at the juncture of pressure waveform segment $i$ and $i - 1$

– $\lambda_i$ the time duration of pressure waveform segment $i$. 
A system of three ordinary differential equations, one for each parameter, is solved to propagate the pressure wave in the atmosphere:

\[
\begin{align*}
\frac{dm_i}{dt} &= C_1 m_i^2 + C_2 m_i, \\
\frac{d\Delta p_i}{dt} &= \frac{1}{2} C_1 \Delta p_i (m_i + m_{i-1}) + C_2 \Delta p_i, \\
\frac{d\lambda_i}{dt} &= -\frac{1}{2} C_1 (\Delta p_i + \Delta p_{i+1}) - C_1 m_i \lambda_i,
\end{align*}
\]

with notations:

\[C_1 = \frac{\gamma + 1}{2\gamma} \frac{1}{\rho} \quad \text{and} \quad C_2 = \frac{1}{2} \left( \frac{1}{a} \frac{da}{dt} + \frac{1}{\rho} \frac{d\rho}{dt} - \frac{1}{A} \frac{dA}{dt} \right),\]

where we denote by \(a\) the air ambient sound speed, \(\rho\) the air ambient density, \(p\) the air ambient pressure and \(A\) the (acoustic) ray tube area as cut by the waveform. All these quantities are functions of the altitude. Here, the wind velocity is assumed to be zero and we use the ICAO Standard atmosphere (see ICAO, 1993).

To solve this system, we just have to advance in time System [3] with a time step sufficiently small for neglecting the variations of \(C_1\) and \(C_2\).

### 3.2. Coupling CFD and wave propagation

We denote by \(L\) the chord-length of the aircraft. The extraction line at a distance 0, \(Ox\), is the line parallel to the flow direction going through the nose of the aircraft. By a downward vertical translation along the Mach cone at a distance \(R\) we get the extraction line at distance \(L\). In practice, the distance from the aircraft is generally

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\[\text{Figure 1. Left, sonic boom problem modeling. Right, illustration of the three parameters } m_i, \Delta p_i \text{ and } \lambda_i \text{ characterizing the pressure wave.}\]
characterized by the ratio $R/L$. We denote by $p$ the acoustical pressure and $p_\infty$ the atmospherical pressure.

The pressure distribution obtained under the aircraft in the near field region is used to set up the initial conditions for the propagation of the acoustic wave to the ground. However, the modeled flow in the near field is three-dimensional and non-linear in nature whereas the propagation is a linear one-dimensional model. Thus, to ensure a valid coupling, the near field solution must be locally axi-symmetric with respect to axis $Ox$ in the vicinity of the line where the pressure distribution is extracted. This is a necessary condition for accurately taking all the elements of the aircraft geometry (body, wings,...) into account. Choosing $R/L$ too small results in an error due to insufficient model matching. And choosing large $R/L$ is difficult because it requires huge three-dimensional meshes and if the numerical solution is propagated with a too important numerical then it will reduce the accuracy of the near field signal. We address the latter issues by applying an anisotropic mesh adaptation strategy.

4. Mesh adaptation model

Mesh adaptation provides a way of controlling the accuracy of the numerical solution by modifying the domain discretization according to size and directional constraints. It is well known that mesh adaptation captures accurately shocks issued from the aircraft in the computational domain while reducing significantly the cpu time, see for instance (Frey et al., 2005). Moreover, anisotropic mesh adaptation reduces significantly the flow solver diffusion allowing shock waves to be propagated accurately far from the jet.

4.1. Anisotropic mesh adaptation

For stationary problems, the mesh adaptation scheme aims at finding a fixed point for the mesh-solution couple. In other words, the goal is to converge towards the stationary solution of the problem and similarly towards the corresponding invariant adapted mesh.

At each stage, a numerical solution is computed on the current mesh with the Euler flow solver and has to be analyzed with an error estimate. The considered error estimate aims at minimizing the interpolation error in norm $L^p$ over the whole domain. From the continuous metric theory in (Alauzet et al., 2006; Leservoisier et al., 2001), an analytical expression of the optimal metric is exhibited that minimizes the interpolation error in $L^p$ norm. This anisotropic metric is a function of the Hessian of the solution which is recovered from the numerical solution by a double $L^2$ projection. This metric will replace the Euclidean one to modified the scalar product that underlies the notion of distance used in mesh generation algorithms. Next, an adapted mesh is generated with respect to this metric where the aim is to generate a mesh such that
all edges have a length of (or close to) one in the prescribed metric and such that all elements are almost regular. Such a mesh is called a unit mesh. The tetrahedral volume mesh is adapted by local mesh modifications of the previous mesh (the mesh is not regenerated) using the following operations: vertex insertion, edge and face swap, collapse and node displacement. The vertex insertion procedure uses an anisotropic generalization of the Delaunay kernel (Frey et al., 2000). Finally, the solution is linearly interpolated on the new mesh. This procedure is repeated until the convergence of the couple mesh-solution is achieved.

4.2. Metric construction.

The notion of length in a metric space is closely related to the notion of metric and subsequently to the definition of the scalar product in the vector space. A metric is a $n \times n$ symmetric definite positive matrix, where $n$ is the space dimension. When this metric is continuously defined over the whole domain, it is called a continuous metric.

Let $u$ be an analytic solution defined on a bounded domain $\Omega$ and let $N$ denotes the desired number of vertices for the mesh. We aim at creating the “best” mesh $\mathcal{H}$, i.e., to find the optimal continuous metric $\mathcal{M}$, that minimizes the interpolation error $(u - \Pi_h u)$ in $L^p$ norm with $N$ vertices. $\Pi_h u$ denotes the linear interpolate of $u$ on $\mathcal{H}$. To this end, a model of the interpolation error with respect to a metric $\mathcal{M}$, denoted $e_{\mathcal{M}}$, is required.

In (Alauzet et al., 2006), a model of the interpolation error for a metric $\mathcal{M}$ is given. It has been proved that locally the optimal metric has for main directions the eigenvectors of the Hessian of $u$. Therefore, the point-wise local error model for such metric in the neighborhood of a vertex $a$ could be simplified to:

$$e_{\mathcal{M}}(a) = \sum_{i=1}^{n} h_i^2 \left| \frac{\partial^2 u}{\partial \alpha_i^2} \right|,$$

where $h_i$ and $\frac{\partial^2 u}{\partial \alpha_i^2}$ stand for sizes prescribed by the metric and the eigenvalues of the Hessian in the direction of the $i^{th}$ eigenvectors of the Hessian, respectively. Now, we are looking for the function $\mathcal{M}$ that minimizes, for a given number $N$ of vertices, the $L^p$ norm of this error. To this end, we have to solve the following problem:

$$\min_{\mathcal{M}} \mathcal{E}(\mathcal{M}) = \min_{\mathcal{M}} \int_{\Omega} (e_{\mathcal{M}}(x))^p \, dx = \min_{\mathcal{M}} \int_{\Omega} \left( \sum_{i=1}^{n} h_i^2(x) \left| \frac{\partial^2 u}{\partial \alpha_i^2}(x) \right| \right)^p \, dx,$$

under the constraint:

$$\mathcal{C}(\mathcal{M}) = \int_{\Omega} \prod_{i=1}^{n} h_i^{-1}(x) \, dx = \int_{\Omega} d(x) \, dx = N.$$
The resulting optimal metric solution of Problem [5] and [6] for the $L^2$ norm in three dimensions reads:

$$M_{L^2} = D_{L^2} (\det |H_u|)^{-\frac{1}{2}} \mathcal{R}_{u}^{-1} |\Lambda| \mathcal{R}_{u}$$

with

$$D_{L^2} = N^2 \left( \int_{\Omega} \prod_{i=1}^{3} \left| \frac{\partial^2 u}{\partial \alpha_i^2} \right| \right)^{-\frac{2}{3}}.$$  

For the sonic boom problem, we have considered the continuous metric controlling the $L^2$ norm of the error as the choice of an $L^p$ norm could be essential in mesh adaptation process regarding the type problem solved. For instance in CFD, physical phenomena can involve large scale variations (e.g. multi-scale phenomena, recirculation, and weak and strong shocks). Capturing weak phenomena is crucial for obtaining an accurate solution by taking into account all phenomena interactions in the main flow area. Intrinsically, metrics constructed with lower $p$ norms are more sensitive to weaker variations of the solution whereas the $L^\infty$ norm mainly concentrates on strong shocks.

### 4.3. Adaptive sonic boom numerical simulation

We consider a supersonic aircraft flying at a supersonic speed of 1.6 Mach with an angle of attack of 3 degrees at an altitude of 13,680 meters (45,000 feet). The aircraft geometry is the supersonic business jet geometry (SSBJ) of Dassault Aviation (Figure 2 left). The length of the SSBJ is 36 meters. The complete aircraft is included in a sphere with a diameter of 1 kilometer, cf. Figure 2 right.

As regards mesh adaptation, we choose to control the error on the Mach number, as

![Figure 2](image-url). Left, SSBJ’s geometry represented by its surface mesh. Right, spherical computational domain with the SSBJ.
the Mach number is really representative of the flow even if an accurate near field pressure distribution is required. A total number of 15 iterations of adaptation has been performed, each 150 time steps of the flow solver. We try to obtain the best mesh controlling the error in $L^2$ norm with 600,000 vertices.

The final adapted anisotropic mesh contains almost 570,000 vertices and 3.3 million tetrahedra, views of this mesh is shown in Figure 3. We notice that mesh refinement along Mach cones have been propagated in the whole computational domain. Such meshes reduce significantly the numerical diffusion introduced by the flow solver. Consequently, the solution, e.g. Mach cones, is accurately propagated in the whole domain, Figure 4.

Indeed, a further analysis shows that the near field signal is propagated accurately until $R/L = 6.5$, Figure 5. According to Whitham or linear supersonic theory a signal is accurately transported in the domain and has not been diffused if the quantity $\sqrt{\frac{p}{p_\infty}} \left(\frac{p}{p_\infty}\right) - p_\infty$ for the two distances are close to each other. This is the case for signals between $R/L = 4$ and $R/L = 6.5$. After, the signal is slightly diffused but it is still well represented. This is clearly illustrated on sonic boom signatures where almost the same signal is obtained for $R/L$ between 4 and 6.5, Figure 5.

In conclusion, for this geometry the signal seems to be converged, and thus the coupling is assume to be valid, for an $R/L$ around 5 with an accuracy of 600,000 vertices.

5. A numerical platform

Our optimization approach consists in translating first into a globally numerical model the following ingredients:

![Figure 3. View of the final anisotropic mesh obtained with $L^2$ norm continuous metric-based mesh adaptation on the Mach number.](image-url)
Multi-model multi-scale optimization

– a discrete shape definition,
– a CFD flow model,
– a functional.

To this end, we build discrete sensitivities, \textit{i.e.}, we apply the Optimal Control theory directly to the discretized equations. The discrete gradients are computed by generating sensitivities by means of the Automatic Differentiation tool TAPENADE developed at INRIA (Hascoët \textit{et al.}, 2004). It provides efficient and more accurate gradients evaluation than the ones obtained by finite differences methods. This approach always guarantees a descent direction whatever the mesh size compared to continuous gradient techniques where a descent direction is only guaranteed when the mesh size converges toward zero. A feature of the presented optimization process lies in the fact that CAD parametrization has been embedded in the loop. Therefore the total set \( \mu = (l, \nu) \) of design variables in the subsequent equations has been split into two parts: \( l \) describes aerodynamic design variables such as angle of attack, side slip angle etc., whereas \( \nu \) stands for geometric design variables handled directly by the CAD modeler. Schematically the CAD modeler can be represented by the following operator:

\[ \nu \mapsto d(\nu) \]

where \( d(\nu) \) denotes the surface mesh displacement. The mathematical optimization problem can be formulated as an optimal control problem (Lions, 1971) as follows (Dinh \textit{et al.}, 1996):

\textbf{Find a shape } \( \mu^* \) \textbf{within } \( \mathcal{O} \) \textbf{ (set of shapes) such that :}

\[ \begin{cases} 
\mu^* = \arg \min_{\mu \in \mathcal{O}} j(\mu) \\
g(\mu) \leq 0 
\end{cases} \]

\textbf{Figure 4. Final solution on the adapted mesh. Left, mach number iso-value in the symmetry plane. Right, Mach number iso-surface representing Mach cones emitted by the aircraft.}
Figure 5. Top, near field signature extracted from several R/L. Bottom, sonic boom signature obtained from different R/L near field initializations. Only choosing R/L more than 5 produces a converged output.
The cost function \( j(\mu) = J(\mu, W(\mu)) \) and constraints \( g(\mu) = G(\mu, W(\mu)) \) depend on the set of design variables \( \mu \) and on the solution vector \( W(\mu) \) of the state equations \( E(\mu, W(\mu)) = 0 \). By assembling cost and constraint functions in one vector, we can write:

\[
\begin{align*}
\mathbf{f}(\mu) &= \mathbf{F}(\mu, W(\mu)) = (J(\mu, W(\mu)), G(\mu, W(\mu)))
\end{align*}
\]

In order to solve the optimization problem at hand, the function \( \mathbf{f}(\mu) \) must be evaluated for variations in the set of design variables denoted \( \delta \mu \).

The mesh global deformation, resulting from the surface mesh deformation \( d(\nu) \), computed by the geometric modeler is contained within \( D(\nu) \) and is obtained by solving an elliptic problem through the operator \( \mathbf{L}(d(\nu), D(\nu)) \). Then, the vector \( \mathbf{f} \) can be rewritten as:

\[
\begin{align*}
\mathbf{f}(\mu) &= \mathbf{F}(l, d(\nu), W(l, d(\nu)))
\end{align*}
\]

The gradient computation consists in expressing the variations of cost and constraints functions with respect to \( \delta \mu \):

\[
\begin{align*}
\delta \mathbf{f} &= \frac{\partial \mathbf{F}(l, D(\nu), W(l, D(\nu)))}{\partial l} \delta l + \frac{\partial \mathbf{F}(l, D(\nu), W(l, D(\nu)))}{\partial D(\nu)} \frac{\partial D(\nu)}{\partial \nu} \delta \nu + \frac{\partial \mathbf{F}(l, D(\nu), W(l, D(\nu)))}{\partial W(l, D(\nu))} \left( \frac{\partial W(l, D(\nu))}{\partial l} \delta l + \frac{\partial W(l, D(\nu))}{\partial D(\nu)} \frac{\partial D(\nu)}{\partial \nu} \delta \nu \right) \quad [8]
\end{align*}
\]

using the state equations:

\[
\begin{align*}
\mathbf{E}(l, D(\nu), W(l, D(\nu))) &= 0
\end{align*}
\]

And the equation for the mesh deformation:

\[
\mathbf{L}(d(\nu), D(\nu)) = 0
\]

For gradient computation in direct mode the following linear system needs to be solved:

\[
\begin{bmatrix}
\frac{\partial \mathbf{E}}{\partial \mathbf{W}} \\
\frac{\partial \mathbf{W}}{\partial l} & \frac{\partial \mathbf{W}}{\partial \nu}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \mathbf{W}}{\partial l} & \frac{\partial \mathbf{W}}{\partial \nu}
\end{bmatrix}
= - \begin{bmatrix}
\frac{\partial \mathbf{E}}{\partial \mathbf{D}} & \frac{\partial \mathbf{E}}{\partial \nu}
\end{bmatrix} \\
\frac{\partial \mathbf{D}}{\partial \nu}
\]

[9]

The operator \( \frac{\partial \mathbf{E}}{\partial \mathbf{D}} \) is obtained by differentiating the Euler flux with respect to the mesh coordinates. Writing these coordinates as \( \mathbf{X}(\nu) \) and taking \( \mathbf{X}_0 = \mathbf{X}(0) \) as the reference mesh coordinates, we have:

\[
\mathbf{X}(\nu) = \mathbf{X}_0 + D(\nu)
\]

Differentiation of the mesh deformation operator has been performed using the automatic differentiation tool in direct mode. Subsequently, the following linear system has to be solved:

\[
\frac{\partial \mathbf{L}(d(\nu), D(\nu))}{\partial D} \frac{\partial D}{\partial \nu} = - \frac{\partial \mathbf{L}(d(\nu), D(\nu))}{\partial d} \frac{\partial d}{\partial \nu} \quad [10]
\]

The RHS of Equation [10] includes the term \( \frac{\partial d}{\partial \nu} \) provided by the geometric CAD modeler. Every single term of the linear System [9] is then determined.
Referring to Equation [8], the gradients of cost and constraints functions $f(\mu)$ are then obtained in direct mode for the Euler equations and mesh deformation by:

$$
\begin{align*}
\frac{dF}{dl} &= \frac{\partial F}{\partial l} + \frac{\partial F}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial l}, \\
\frac{dF}{d\nu} &= \frac{\partial F}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} + \frac{\partial F}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu}.
\end{align*}
$$

[11]

In our problem, the number of design variables is greater than the number of cost and constraint functions to be evaluated. In that case, regarding the number of linear systems to solve, it is advantageous to work with a system using two adjoint variables. Let us first introduce an adjoint related to the state equation. One defines $\Psi$ as the solution of:

$$
\begin{bmatrix}
\frac{\partial E}{\partial \mathbf{W}}
\end{bmatrix}^T \Psi = \begin{bmatrix}
\frac{\partial F}{\partial \mathbf{W}}
\end{bmatrix}^T.
$$

[12]

Next, an adjoint $\Phi$ related to the mesh deformation is computed by:

$$
\begin{bmatrix}
\frac{\partial L}{\partial \mathbf{D}}
\end{bmatrix}^T \Phi = \begin{bmatrix}
\frac{\partial F}{\partial \mathbf{D}}
\end{bmatrix}^T - \begin{bmatrix}
\frac{\partial E}{\partial \mathbf{D}}
\end{bmatrix}^T \Psi.
$$

[13]

The variations of cost and constraints functions are then computed using $\delta f = \delta f - \Psi^T \delta E - \Phi^T \delta L$:

$$
\begin{align*}
\delta f &= \frac{\partial F}{\partial l} \delta l + \frac{\partial F}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} \delta \nu \\
&\quad + \frac{\partial F}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial l} \delta l + \frac{\partial F}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} \delta \nu \\
&\quad - \Psi^T \frac{\partial E}{\partial l} \delta l - \Psi^T \frac{\partial E}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} \delta \nu \\
&\quad - \Psi^T \frac{\partial E}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial l} \delta l - \Psi^T \frac{\partial E}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} \delta \nu \\
&\quad - \Phi^T \frac{\partial L}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} \delta \nu - \Phi^T \frac{\partial L}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} \delta \nu.
\end{align*}
$$

[14]

Equation [14] allows us to calculate the gradients in adjoint mode for the state equation and mesh deformation as:

$$
\begin{align*}
\frac{dF}{dl} &= \frac{\partial F}{\partial l} - \Psi^T \frac{\partial E}{\partial l}, \\
\frac{dF}{d\nu} &= -\Phi^T \frac{\partial L}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} - \Phi^T \frac{\partial L}{\partial \mathbf{D}} \frac{d\mathbf{D}}{d\nu} \delta \nu.
\end{align*}
$$

[15]

The gradients of cost and constraints functions $f(\mu)$ have been obtained by solving the Systems [12] and [13] where the number of RHS terms does not depend on the
number of design variables but is directly related to the number of cost and constraints functions. For this reason the adjoint approach is used in the majority of industrial optimization problems.

5.1. **Optimization platform**

The optimization platform couples the following modules:

- CAD modeler
- Volume mesh deformation
- CFD solver (Bastin *et al.*, 1999)
- Adjoint of CFD solver (Dinh *et al.*, 1996)
- Adjoint of volume-mesh deformation
- Cost and gradient
- Optimizer.

The optimizer has a crucial contribution in the optimization process. Indeed, this tool will drive the whole process by analyzing different values of the cost function and the related constraints and their sensibility with respect to the design parameters. The outputs of the optimizer are a new set of parameter values for which it is necessary to provide: the cost function and the associated constraints along with their derivatives. In the current study, we use the optimizer developed at Dassault Aviation. This code is based on gradient evaluations which should be calculated either by finite differences or by adjoint formulation as explained later. The different available optimizers are Broydon-Fletcher-Goldfarb-Shanno (BFGS) for unconstrained cases and the Method of Feasible Directions, a novel version of Interior Point Algorithm (Herskovits *et al.*, 1996), or Sequential Quadratic Programming for constrained cases.

5.2. **CAD modeler**

The CAD modeler named GANIMEDE (Geometry ANd Inherent MEsh DEformation) handles both local and global design variables. By local design variables at control point, we mean position, tangent and curvature values. Global design variables redefine several control points enabling to modify characteristics such as thickness, twist and camber of wing sections.

The geometry itself is decomposed by its definition in a hierarchical way. Geometric entities are defined by a set of patches specified by the user. These entities define a set of sections which in turn are defined by a number of control points. Global design variables can act at section or entity level of the hierarchical model whereas local design variables can act at all levels. Therefore, a variable can control a more or less larger part of the aircraft and thereby allowing for a flexible parametrization of the geometry. The decomposition in entities and sections of a CAD model is shown in Fi-
Surface patches define a polynomial function of degree \( N_{\text{deg}} \) in the \( uv \) space. For a given set of design variables \( \nu \) this can be written as:
\[
S(u, v, \nu) = \sum_{(i,j) \in \{0,\ldots,N_{\text{deg}}\}^2} u^{N_{\text{deg}}-i} v^{N_{\text{deg}}-j} c_{i,j},
\]
where \((u, v) \in [0, 1]^2\) and \((c_{i,j})_{i,j}\) represent the coefficients of the polynomial function. The different geometric entities specified by the user might intersect and the design variables might have an impact on the intersection. The geometric modeler updates the new intersection by computing:
\[
\sum_{(i,j) \in \{0,\ldots,N_{\text{deg}}\}^2} u^{N_{\text{deg}}-i} v^{N_{\text{deg}}-j} c_{i,j} = \sum_{(i,j) \in \{0,\ldots,N_{\text{deg}}\}^2} \sigma^{N_{\text{deg}}-i} \tau^{N_{\text{deg}}-j} \rho_{i,j},
\]
where \((u, v)\) and \((\sigma, \tau)\) are the parameters and \((c_{i,j})\) and \((\rho_{i,j})\) the polynomial coefficients of the different patches involved in the intersecting entities.

As the geometry is modified during the optimization process, its corresponding new surface mesh has to be regenerated. The CAD modeler also copes with this aspect as was mentioned previously. To this end, a connectivity is created by projection of the initial surface mesh on the initial geometry. Then, the geometry CAD model is deformed, \(i.e.,\) a new geometry is created, according to design parameters. Finally, from the initial surface mesh and the new CAD geometry, a new iso-topologic surface mesh is generated.

5.3. Mesh deformation

As mentioned previously the CAD modeler can be schematically represented by the operator: \( \nu \mapsto \mathbf{d}(\nu) \) where \( \mathbf{d}(\nu) \) represents the surface mesh displacement resulting from the geometric design variables \( \nu \). This surface mesh displacement is then propagated into the volume mesh by an elliptic operator: \( \mathbf{L}(\mathbf{d}(\nu), \mathbf{D}(\nu)) \), where \( \mathbf{D}(\nu) \) designates the resulting displacement field throughout the volume mesh.

The chosen operator is a Laplacian like operator, more precisely:
\[
\begin{align*}
-\nabla \cdot (\kappa \nabla \mathbf{D}(\nu)) &= 0 \\
\mathbf{D}(\nu) |_{\Gamma_c} &= \mathbf{d}(\nu) \\
\mathbf{D}(\nu) |_{\Gamma_\infty} &= 0,
\end{align*}
\]
where \( \Gamma_c \) represents the shape surface boundary, \( \Gamma_\infty \) the boundary of the computational domain and \( \kappa \) a local coefficient related to the size (volume) of the local tetrahedral element. As the operator is symmetric and positive semi definite, the linear system is solved by incomplete Cholesky preconditioning in addition to a preconditioned conjugate gradient (PCG) method. In order to improve robustness of the mesh deformation, the surface mesh displacement can be performed in several steps. At each step, the surface displacement is propagated into the surrounding volume mesh. This process has been parallelized by splitting up the computational domain in several domains and using the MPI library as communication protocol between processors.
5.4. Cost and constraints

In the current optimization process, available cost and constraints functions are based on the six global aerodynamic coefficients (pressure drag, lift, pitching moment, ...), on the wing span-wise lift distribution or on the local pressure distribution on the aircraft. Many other functions have been developed to address other problems than sonic boom reduction (like inlet design, engine integration, ...). Due to the difficulty to differentiate an accurate wave propagation tool, our choice is to develop a functional based on near field deviation of pressure fluctuation to a target one.

Figure 6. Aircraft simplified geometry for sonic boom reduction. Top left, decomposition of geometry CAD model. Top right and bottom, comparison between baseline and optimized geometries.
Let line \([l_{θ_1}, l_{θ_2}]\) be the extraction line at distance \(R\) characterized by the angle of attack \(θ\), see Figure 7. We define the cost function related to angle \(θ\) as

\[
f_θ(W) = \frac{1}{2} \int_{l_{θ_1}}^{l_{θ_2}} \left[ dp_θ(W) - dp_{θtarget}(W) \right]^2 dl,
\]

with \(dp(W) = (p(W) - p_∞) / p_∞\). If we have to deal with a list of angles \((θ_i)_{i=1,N}\), we propose to realize a linear combination of cost functions \((f_{θ_i})_{i=1,N}\). Thus, the global cost functional becomes:

\[
f(W) = \sum_{i=1,N} \omega_i f_{θ_i}(W),
\]

with \((ω_i)_{i=1,N}\) weights associated to each angle \(θ_i\).

### 5.5. Sensitivity development and optimizer

The TAPENADE automatic differentiation software acts by transformation of the initial code. Having a given set of input variables and a group of programs written in Fortran 77 evaluating the numerical function \(f\), TAPENADE generates sub-programs computing its derivatives with respect to those variables. This software allows us to use direct or reverse mode of automatic differentiation. Reverse mode is applied to get routines computing adjoint residual and functional gradient. See (Hascoët et al., 2004) for further details.

### 5.6. A numerical example

#### 5.6.1. Shape optimization

Sonic boom reduction is addressed with the mean field target pressure method. The target pressure is set to far field pressure. Then, the platform is applied to the inverse problem of finding the shape for which the flow matches at best the target mean field pressure while maintaining the lift coefficient equal to the baseline value and keeping drag coefficient lower or equal to the baseline value. A cost functional is then defined as the mean square of deviation between current and target pressure on an horizontal plane at \(R/L = 0.5\). The baseline geometry is a wing-body combination involving a generic fuselage (cone-cylinder-cone) and a high-sweep wing, Figure 6. In addition to flow constraints, the following geometrical constraints are applied: length and wing thickness are kept to baseline values, and cabin has to contain a certain volume for passengers, \(i.e.,\) its section for a fixed \(x\)-interval is larger than the baseline section. The shape parameters involve 55 CAD fuselage parameters and 11 wing parameters. The 67th parameter is the angle of attack. A volume mesh containing 163, 459 vertices is used to compute the flow.
Figure 8 shows the evolution of cost function and constraints as functions of optimizer iterations, each iteration of the interior point algorithm involving one cost evaluation and one gradient evaluation. The cost function is divided by 2 in less than 60 iterations. The two main shape modifications concern the nose tilting and the wing bending, Figure 6. The effect of shape modifications on shocks emitted from the aircraft are clearly illustrated in Figure 7 where iso-values of pressure are represented in the symmetry plane. The consequence of the optimization on the sonic boom signature at ground, as given by the propagation post treatment, is a reduction almost 20 % (7 Pascal) of the initial pressure rise, see Figure 8. We observe a smaller shock focalization rather that a weaker shock system. Maximal pressure is lower and minimal pressure is higher.

5.6.2. Validation with mesh adaptation

In the previous section, automatic shape optimization has been carried out on a relatively coarse non-adapted mesh. Consequently, the pressure signal used in the cost function definition has been extracted close to the aircraft at \( \frac{R}{L} = 0.5 \). Indeed, the numerical dissipation on such meshes becomes more and more important when we go away from the aircraft. This implies a quasi-nul pressure signal if the extraction is performed too far from the jet. Nonetheless, we shown in Section 4.3 that larger \( \frac{R}{L} \) are required to have a valid coupling between the CFD computation and the propagation. Therefore, we now want to validate gains obtained with the optimization process on the coarse mesh by studying finely the flow associated with shapes before and after optimization. To this end, accurate adaptive simulations are performed on both

Figure 7. Sonic boom : view of near field pressure. Left, initial shape and right, optimized shape. The line under the aircraft is the pressure extraction line to evaluate the cost function.
shapes to get a reliable pressure signal far from the aircraft, as described in Section 4. The parameters of Section 4.3 have been chosen for the adaptive simulations. Only, a slight modification of the angle of attack has been done in order to have the same lift coefficient for both simulations.

For each simulation, an accurate signal is obtained until $R/L = 5$. Near field signals at $R/L = 1$ and $R/L = 5$ are presented Figure 9, and the associated sonic boom signatures are depicted in Figure 10. Figure 9 clearly illustrates the impact of the optimized shape on the near field pressure distribution. With this new shape, all shocks are splitted into several ones with a reduced intensity. Nevertheless, if the propagation from the signals at $R/L = 1$ shows the same tendency with a reduced impact as compared to the results obtained at $R/L = 0.5$ on the coarse mesh, the results of the propagation from $R/L = 5$ are more disappointing. In this case, both shapes almost produce the same sonic boom signature.

Several ways of improving the optimisation are envisaged:

– to include the mesh adaptation process inside the optimization loop to have an accurate near field evaluation at a sufficiently far distance from the aircraft. Then, the hypothesis for the coupling between the CFD and the propagation model will be verified

– to include the propagation scheme inside the optimization loop. But, it requires to have a differentiable propagation code.

In the following section, the first approach is considered.

6. Coupling mesh adaptation and shape optimization

In the previous section, the objective functional has been decreased on a fixed non-adapted mesh. We consider now the research of an optimum in combination with the mesh adaptation algorithm. In other words, we want to get a shape that is optimal

![Figure 8.](image)

Figure 8. Right, functional and constraints evolution during minimization. Right, sonic boom reduction at ground level.
when the objective function is evaluated on a mesh that is strongly adapted to the optimal flow. As the optimal shape is not known, the associated adapted mesh cannot be generated in advance. Adapted mesh has to be constructed at the same time we optimize. As a consequence, we cannot define a stand alone discrete optimization problem as done in the previous section. Instead, we propose to approximatively solve the continuous optimality condition with a mesh adaptive algorithm. This could be done by relaxing the two non-linear effects:

- minimizing the functional for a fixed mesh,
- generating an adapted mesh in a fixed domain.

**Figure 9.** Near field pressure distributions at $R/L = 1$ (left) and $R/L = 5$ (right) for the initial and the optimized shapes obtained with adaptive simulations.

**Figure 10.** Sonic boom signatures propagated from pressure signals, obtained with adaptive simulations, at $R/L = 1$ (left) and $R/L = 5$ (right) for the initial and the optimized shapes.
However, the formulation must take care of the existence of a fixed point for both effects.

6.1. Minimizing for a fixed mesh

As a steepest descent algorithm is applied, we need to identify which part of the algorithm cannot be successfully achieved when the mesh is modified. Our option is still to use an exact gradient approach in order to keep a reliable descent direction. Then, the following sequence is applied with a fixed mesh:

\textit{Gradient and line search:}
- compute the flow (state equation)
- compute the adjoint state
- compute the (exact) gradient of functional
- line search in the descent direction.

In order to keep an invariant mesh during that sequence, the shape variation is performed without changing the computational domain, thanks to transpiration boundary conditions.

6.2. Mesh adaptation for a fixed domain

A mesh that is accurately adapted to a flow will be much less accurate when used for computing an -even slightly- different flow. Starting a line search with a mesh adapted to the first flow may then result in poor evaluation of the other flows and a poor evaluation of the descent step length. To avoid this, we proposed a specific algorithm based on the fixed point mesh adaptation method introduced in (Alauzet \textit{et al.}, 2007).

This scheme enable us to generate meshes adapted for different shapes during the gradient and line search procedure, \textit{i.e.}, a mesh adapted to each flow associated with each shape. More precisely, in the fixed point mesh adaptation/gradient loop, the mesh is adapted to the \( k \)-th gradient as well as to the search step by taking into account all solutions computed throughout this step:

- to each flow corresponds an optimal metric
- the \textit{intersection} of all these metrics is computed
- the adapted mesh is generated from the resulting intersected metric.

Notice that the adapted mesh cannot be generated before the evaluation of the flow variables. It means that an implicit coupling needs to be applied. We address this issue with an iterative process which aims at converging to a fixed point:

\textit{Fixed point adaptation/gradient step:}

1) choose an initial mesh
2) compute on current mesh the flow (state equation)
3) compute on current mesh the adjoint state
4) compute on current mesh the (exact) gradient of functional
5) perform on current mesh line search in the descent direction
6) compute the intersection of metrics for all flows in steps 2-5, and generate a new mesh specified by the new metric
7) if process not converged, go to 2.

The process is considered converged in step 7 as soon as the difference between two consecutive metrics is small. In practice, this fixed point loop iterates about 5 times to converge. Between each remeshing, the computed solution on the previous mesh (step 2) is transferred on the new adapted mesh to reduce computing expenses. The fixed point adaptation/gradient step is then itself included in the gradient loop.

It is necessary to initially set the desired accuracy for the solution in the fixed point adaptation process to have a well-posed problem with respect to the metric. The number of nodes and the solution may change but the prescribe size is always consistent along the process to keep the accuracy of the solution at the required level. However, when the user wishes to increase this number of nodes from one gradient step to another, some clever strategy is necessary.

6.3. Application to problem under study

Preliminary optimization computations have been applied to an HISAC test case. The shape is optimized in the same conditions as for the previous sections, but here the shape parametrization is CAD-free and shape constraints are not applied. An adapted mesh sample used during the adaptive optimization platform is shown in Figure 11, left.

Figure 11 (right) depicts the evaluation of functional during the coupled loop. The oscillations observed in the functional curve are associated to the mesh adaptation phase which is devoted to find the best adapted mesh and then ensure the good evaluation of the functional. This mesh adaptation influence over global optimization loop gives us a computation certainty along the optimization cycle.

Figure 12 points out the reduction obtained with the adaptive optimization platform for the near field pressure signal at $R/L = 1$ and the sonic boom signature after propagation.

7. Conclusions

The presented work shows the complexity of an optimization platform for aero-dynamics and sonic boom. Automatic Differentiation is used for developing the exact sensitivity of an industrial chain involving CAD parametrization, mesh deformation,
complex flow model. The strong coupling between a new accurate mesh adaptation algorithm and the optimization platform has been also studied.

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9. Bibliographie


Thomas C., Extrapolation of sonic boom pressure signatures by the waveform parameter method, TN, n° D-6832, Nasa, 1972.


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