Flow simulation in 3D Discrete Fracture Networks (DFN)

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Fracture Structures
Scales, Organization and Diversity

Granite (Sweden) 1 m

Coal (Australia) 5 cm

Sandstones (Norway) 1 km

Shale (US) 1 m

Sandstones (Norway) 1 km
Fracture reconstruction
DFN models

Widely-distributed fracture lengths
Stress-limited connectivity

\[ a = 3.5, \quad p = 1.2, \quad U_{\text{min}} = 10, \quad \text{Poissonian} \]

Single-Phase flow

Assumptions
- Steady-state
- Only in fractures, impervious rock matrix

Flow equation in each fracture \( \gamma \):
\[
\begin{align*}
\nabla \cdot \mathbf{u}(x) &= f(x), & & \text{for } x \in \gamma, \\
\mathbf{u}(x) &= - (x)\nabla \rho(x), & & \text{for } x \notin \gamma, \\
\rho(x) &= \rho^0(x), & & \text{on } \Gamma_0 \cap \Gamma_\gamma, \\
\mathbf{u}(x) \cdot \mathbf{n} &= q^q(x), & & \text{on } \Gamma_N \cap \Gamma_\gamma, \\
\mathbf{u}(x) \cdot \mathbf{n} &= 0, & & \text{on } \Gamma_\gamma \setminus ((\Gamma_\gamma \cap \Gamma_0) \cup (\Gamma_\gamma \cap \Gamma_N))
\end{align*}
\]

\( \mathbf{v} \) (resp. \( \mu \)) outward normal unit vectors

\( T(x) \): transmissivity field \([m^2.s^{-1}]\)

\( f(x) \): sources/sinks

Continuity conditions at intersections \( \Sigma \):
\[
\sum_{\gamma \in \Gamma_k} \mathbf{u}_{k,\gamma} \cdot \mathbf{n}_{k,\gamma} = 0
\]

On \( \Sigma_k \) intersection between fractures \( \gamma \) of \( K_k \)
Computational domain

Fracture Statistics

Multiple scales of heterogeneity

- Power-law length distribution \( n(l) \sim l^{-a} \) (2≤a≤4)
- Stress-induced correlations

Fracture density

- Heterogeneous and "increasing" with scale
- \( 10^3 \) to \( 10^5 \) fractures

Fracture apertures

- Self-affine truncated Gaussian distribution
- Fracture Transmissivity \( T \sim a^3 \)

Fracture orientations

- Orthogonal
- Fisher or uniform distributions

Secondary correlations

- Aperture-Length
- Aperture-Orientation
- Length-Density

Numerical Challenges of DFNs

Multi-Scale, Robust, Efficient

Multiple scales of heterogeneity

- Few large fractures
- Bulk contribution of small fractures
- 1D channels, 2D fractures, 3D space

Large domains

- \( 10^3 \) to \( 10^5 \) fractures

High heterogeneity

- Widely distributed transmissivities
- Topology close to some critical state

Weak constrains for high complexity

- Stochastic modelling
Numerical Challenges of DFNs  
*Multi-Scale, Robust, Efficient*

- **Multiple scales of heterogeneity**
  - Few large fractures
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  - 1D channels, 2D fractures, 3D space

- **Large domains**
  - $10^3$ to $10^5$ fractures

- **High heterogeneity**
  - Widely distributed transmissivities
  - Topology close to some critical state

- **Weak constraints for high complexity**
  - Stochastic modelling

- **Intricate local configurations**
  - Local clustering of fractures
  - Fracture intersecting by their tips

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Fracture-Network Decomposition Method  
*Sparse flow structure*

- **2D Discretization**
  - Fracture boundaries and intersections
  - Removes the locally intricate configurations
  - Remains local to the fracture

- **Mesh generation**
  - 2D in the fracture plane
  - Standard mesh generation techniques

- **Discretization scheme in fractures**
  - Mixed-Hybrid Finite Element Method

- **Continuity at fracture intersections**
  - Mortar conditions

- **Linear system solver**
  - Domain decomposition

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2D Discretization and Mesh generation

Does the mesh quality matter?

Mesh quality after discretization

Quality mesh criterion $Q_K \in [0;1]$ for each triangle $K$:

$$Q_K = 4\sqrt{3} \frac{S_K}{h_k^2}$$

- $S_K$: surface of $K$
- $h_k$: mean edge length

Optimal triangle quality: $Q_k = 1$

Mesh quality with discretization
Number of triangles: 26,882
Minimum of $Q_K$: 0.51

Mesh quality without discretization
Number of triangles: 18,023
Minimum of $Q_K$: 5.6 $10^{-3}$

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Mixed-Hybrid Mortar method
Non-conforming mesh

For each of the intersections: arbitrary choice of master (m) and slave (s) sides

Mixed-Hybrid Mortar method
Continuity conditions at fracture intersections

Notations

\[
\begin{array}{|c|c|c|}
\hline
\text{Notation} & \text{Local (fracture \( f \))} & \text{Global (network)} \\
\hline
\text{Cell mean hydraulic head} & P_f & P = (P_f)_1 \\
\text{Traces of hydraulic head} & A_f = (A_{m,s})_f & \text{projection from master to slave side} \ \\
& A_{m,s} = (A_{m,s})_f & \\
& A_{m,s} = (A_{m,s})_f & \\
& A_{m,s} = (A_{m,s})_f & \\
& A_{m,s} = (A_{m,s})_f & \\
& A_{m,s} = (A_{m,s})_f & \\
\hline
\text{Jump of flux through the edges} & Q_{m,s} = (Q_{m,s})_f & \\
& Q_{m,s} = (Q_{m,s})_f & \\
& Q_{m,s} = (Q_{m,s})_f & \\
& Q_{m,s} = (Q_{m,s})_f & \\
& Q_{m,s} = (Q_{m,s})_f & \\
\hline
\end{array}
\]

Continuity conditions

Trace of hydraulic head

\[
A_f = CA_m \quad \text{jump of flux} \quad Q_{m,s} + C^TQ_s = 0
\]

\[
C_{m,s} = \left( \frac{E_m \cap E_s}{|E_s|} \right)
\]

\[
P = \left( \begin{array}{c}
R_m \\
R_m \left( A_m + A_C \right) + \left( A_m \right)
\end{array} \right)
\]

Network scale system

\[
\begin{align*}
C_f + R_m & = \left( A_m + A_C \right) \\
M_f & \left( A_m + A_C \right) \\
M_f & \left( A_m + A_C \right)
\end{align*}
\]

\[
\left( A_m \right) - (A_m) - \left( A_m \right) = 1.
\]

\[
A_m - \left( A_m \right) - \left( A_m \right) = 1.
\]

\[
P - \nu = 0.
\]

Mixed-Hybrid Mortar method

Convergence Test


Fracture-Network Decomposition Method

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Advantages
  ► High quality mesh
  ► Decouples fracture and network scales
  ► Avoids global operations before solving
  ► Enables parallelization
  ► Prepares for mesh refinement
Conclusions and perspectives

Optimization
- A posteriori estimators for mesh refinement
- According to flow sparsity

Method combination for DFN flows
- Classic: mesh generation, pde discretization
- Specific: discretization, Mortar

Multi-scale DFNs
- Parallel computation and scalability
- Upscaling rules

Process coupling
- Transport (fracture/matrix)
- Mechanics (fracture/matrix)