# Optimal control techniques based on infection age for the study of the COVID-19 epidemic

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Confinement control in the SIR setting:

$$\begin{aligned}\dot{S}(t) &= -\delta(1-u(t))S(t)I(t)\\ \dot{I}(t) &= \delta(1-u(t))S(t)I(t)-(\eta+\mu)I(t)\\ \dot{R}(t) &= \mu I(t)\end{aligned} \tag{1}$$

with $u(t) \in [0,1]$ confinement control, and $\delta > 0$ infection coefficient, $\eta > 0$ death rate, $\mu > 0$ recovery rate.

## Infection age; related differential calculs

For non hospitalized $z(t,a,b)$ and hospitalized population $h(t,a,b)$
$a$ class of population (strong/weak)
$b \in [0,B]$ infection age;
For $\varepsilon > 0$ small:

$$z(t+\varepsilon, a, b+\varepsilon) \approx z(t,a,b) - \varepsilon \nu(a,b) z(t,a,b) \tag{2}$$

Corresponding differential equation

$$z_t(t,a,b) + z_b(t,a,b) = -\nu(a,b) z(t,a,b) \tag{3}$$

# Full model

States: $y$ susceptible, $z$ infected non hospitalized, $h$ hospitalized, $\bar{y}$ recovered.

$$\begin{cases} \dot{y}(t,a) = -\delta(a)(1-u(t,a))Z(t)y(t,a) \\ (z_t + z_b)(t,a,b) = -\nu(a,b)z(t,a,b) \\ z(t,a,0) = \delta(a)(1-u(t))Z(t)y(t,a) \\ (h_t + h_b)(t,a,b) = \nu(a,b)z(t,a,b) - (\eta(a,b) + \gamma(a,b)E(t))\,h(t,a,b) \\ h(t,a,0) = 0 \\ \dot{\bar{y}}(t,a) = z(t,a,B) + h(t,a,B) \end{cases} \tag{4}$$

## Details of dynamics

$$Z(t) = \int_0^A \int_0^B e(a,b) z(t,a,b) \mathrm{d}a \mathrm{d}b, \tag{5}$$

with $e(a,b)$ transmission factor
hospitalized patients do not contribute to the transmission
$\nu(a,b)$ hospitalization coefficient,
$E(t) \in [0,1]$ is the hospital saturation estimate, given by

$$E(t) := \frac{(H(t) - C)_+}{H(t) + C}; \quad H(t) := \int_0^A \int_0^B h(t,a,b) \mathrm{d}a \mathrm{d}b, \tag{6}$$

where $C > 0$ nominal capacity.

Only hospitalized patients die, with death rate

$$d(t,a,b) := \eta(a,b) + \gamma(a,b)E(t), \tag{7}$$

where $\eta(a,b) \geq 0$ is the minimal death rate and $\gamma(a,b)$ sensitivity of the death rate w.r.t. the hospital saturation estimate.

# Cost function

$$J(M,u,D_T) := p_M M + p_u c(u) + p_D D_T, \tag{8}$$

where the penalty coefficients $p_M$, $p_u$ and $p_D$ are nonnegative.
$D_T$, death toll; $M$ hospital peak value, subject to

$$H(t) \leq M, \quad \forall t \in [0,T]. \tag{9}$$

In addition, in our model we include confinement duration constraints for each class, more precisely:

$$\int_0^T u(t,a)\mathrm{d}t \leq M(a), \quad \text{for a.a. } a \in (0,A). \tag{10}$$

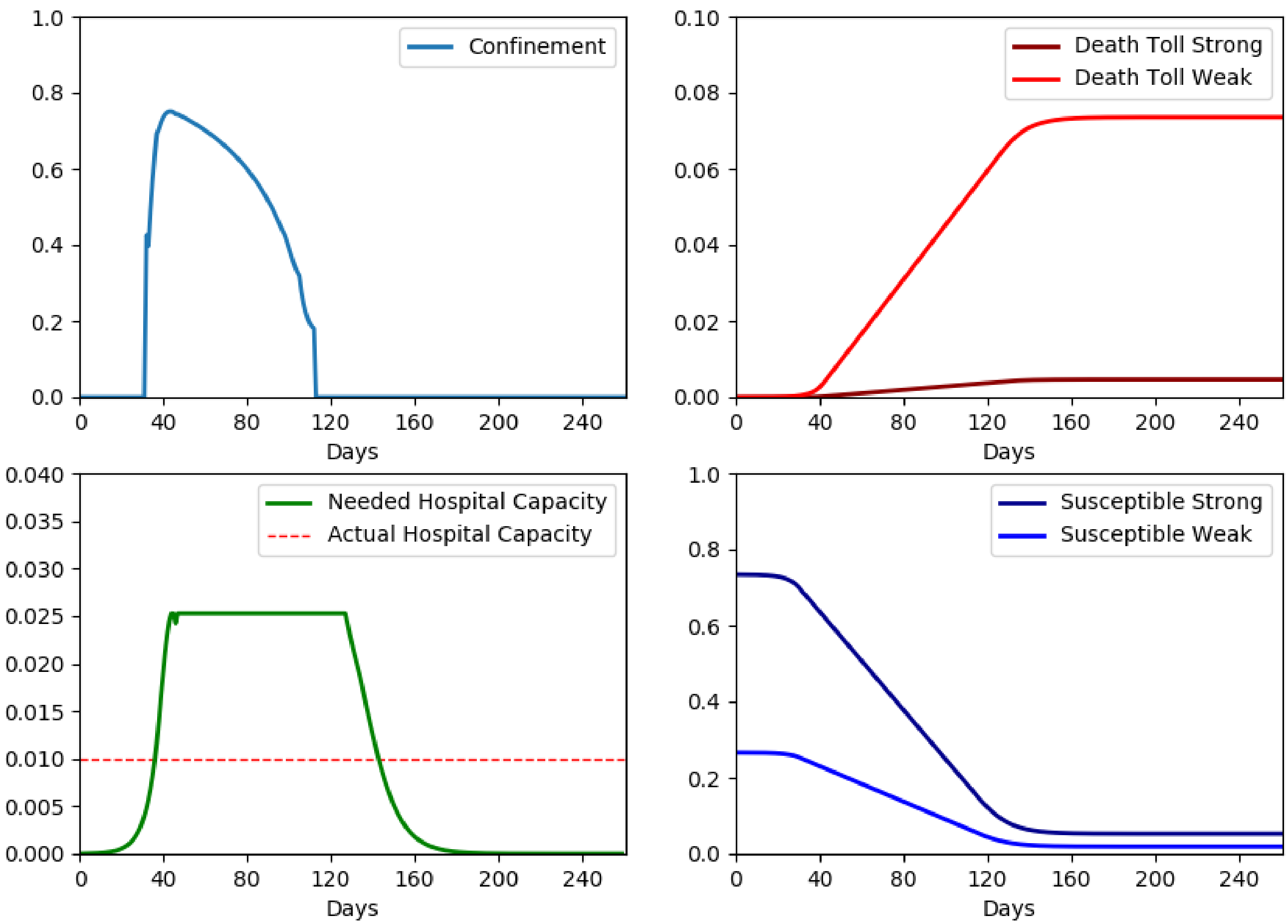


Figure 1: Minimal hospital peak value: $p_M = 10$, $p_u = 0.005$, $p_D = 1$, $C = 0.01$, $T = 260$.